# Sixth Grade Standards: MATHEMATICS

*Note: This information has been adapted from the 2020 Colorado State Standards as presented on the Colorado Department of Education (CDE) website. It is *not* an exhaustive or detailed list. All standards mentioned represent skills grade-level students should have mastered by the end of the grade-level year. If you desire further information, please visit the Standards page on the CDE website: https://www.cde.state.co.us/standardsandinstruction/standards

This document provides tiered support in addressing the academic standards. Families can choose to approach their curriculum selection and content-area instruction in one or all of three categories: a general **Overview** of expectations and "mathematic" behaviors, **Learning Objectives** (a "fly by" glance of concepts a student masters throughout the school year) and **Specific Skills** (expanded ideas to explain the learning objectives). As you consider lesson planning for each grade level, use the "Overview" and "Learning Objectives" checklists to help you plan out your year. Start with the end in mind: If my child needs to know how to _____ by the end of the school year, what learning activities can be implemented to introduce and then reinforce the concepts? Think next about smaller steps in learning that your child needs to master in order to reach that end goal (he/she can't count to 100 if he/she can't first count to ten). Also keep in mind that most objectives are not learned in isolation, meaning learning objectives are often combined. You don't need to ensure your child has mastered learning objective #1 before moving on to the next. Combining two or more objectives in a week's lesson plan can make for more creative and integrated learning. If you are using a reputable and research-based curriculum, then your child will most likely be working his/her way through these learning objectives in a well-paced and consistent manner. (A brief sampling of solid curriculum options can be found on the CSP website under "Homeschool Resources.")

## Overview

**Expectations for 6th Grade Students:**

- **Number and Quantity:** Compare quantities using ratios and unit rates (such as miles per hour); fluently add, subtract, multiply, and divide fractions and decimals; understand the concept of negative numbers and absolute value; extend the number line and coordinate grids to include negative numbers.
- **Algebra and Functions:** Analyze relationships between variables using tables, graphs, and equations; solve one-variable equations and inequalities; apply the order of operations to find the value of an algebraic expression.
- **Data, Statistics, and Probability:** Create graphs including dotplots, boxplots, and histograms; describe data by examining the center (averages) and spread (variability) of a distribution.
- **Geometry:** Apply formulas for the area of triangles and quadrilaterals, including parallelograms and trapezoids; find the volume of rectangular boxes; calculate the surface area of three-dimensional figures.

**Throughout 6th Grade You May Find Students:**

- Finding a cyclist's speed in miles per hour by creating ratio tables, graphs, and number lines.
- Calculating a better deal (buying a gallon or four quarts of milk) using unit prices or ratios.
- Explaining the connection between latitude and longitude on a map and the horizontal and vertical axes on a coordinate grid.
- Recognizing situations involving negative numbers such as temperature, sea level, and bank account balances.

- Designing packaging by creating two-dimensional cutouts and folding them into three-dimensional boxes.
- Describing the difference between the independent and dependent variables for a phone plan.
- Collecting and using data to answer the question: how many hours does the typical sixth-grade student sleep?
- Explaining why average home prices are reported as medians instead of means.

## Learning Objectives

1. Understand ratio concepts and use ratio reasoning to solve problems.

2. Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

3. Compute fluently with multi-digit numbers and find common factors and multiples.

4. Apply and extend previous understandings of numbers to the system of rational numbers.

5. Apply and extend previous understandings of arithmetic to algebraic expressions.

6. Reason about and solve one-variable equations and inequalities.

7. Represent and analyze quantitative relationships between dependent and independent variables.

8. Develop understanding of statistical variability.

9. Summarize and describe distributions.

10. Geometry: Solve real-world and mathematical problems involving area, surface area, and volume.

## Specific Skills

**1. Understand ratio concepts and use ratio reasoning to solve problems.**

- Apply the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was* $2:1$*, because for every* $2$ *wings there was* $1$ *beak." "For every vote Candidate* $A$ *received, Candidate* $C$ *received nearly three votes."*
- Apply the concept of a unit rate $\frac{a}{b}$ associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of* $3$ *cups of flour to* $4$ *cups of sugar, so there is* $\frac{3}{4}$ *cup of flour for each cup of sugar." "We paid $75 for* $15$ *hamburgers, which is a rate of $5 per hamburger." (Expectations for unit rates in this grade are limited to non-complex fractions.)*
- Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

- Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
- Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took* $7$ *hours to mow* $4$ *lawns, then at that rate, how many lawns could be mowed in* $35$ *hours? At what rate were lawns being mowed?*
- Find a percent of a quantity as a rate per $100$ (e.g., $30\%$ of a quantity means $\frac{30}{100}$ times the quantity); solve problems involving finding the whole, given a part and the percent.

- Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.
  - How are ratios different from fractions?
  - What is the difference between a quantity and a number?
  - How is a percent also a ratio?
  - How is a rate similar to and also different from a unit rate?

**2. Apply and extend previous understandings of multiplication and division to divide fractions by fractions.**

- Interpret and compute quotients of fractions and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for* $\frac{2}{3} \div \frac{3}{4}$ *and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that* $\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$ *because* $\frac{3}{4}$ *of* $\frac{8}{9}$ *is* $\frac{2}{3}$*. (In general,* $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$*.) How much chocolate will each person get if* $3$ *people share* $\frac{1}{2}$ *lb of chocolate equally? How many* $\frac{3}{4}$*-cup servings are in* $\frac{2}{3}$ *of a cup of yogurt? How wide is a rectangular strip of land with length* $\frac{3}{4}$ *mi and area* $\frac{1}{2}$ *square mi?*

**3. Compute fluently with multi-digit numbers and find common factors and multiples.**

- Fluently divide multi-digit numbers.
- Fluently add, subtract, multiply, and divide multi-digit decimals.
- Find the greatest common factor of two whole numbers less than or equal to $100$ and the least common multiple of two whole numbers less than or equal to $12$. Use the distributive property to express a sum of two whole numbers $1$–$100$ with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express* $36 + 8$ *as* $4(9 + 2)$*.*

**4. Apply and extend previous understandings of numbers to the system of rational numbers.**

- Explain why positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of $0$ in each situation.
- Describe a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

- Use opposite signs of numbers as indicating locations on opposite sides of $0$ on the number line; identify that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that $0$ is its own opposite.
- Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; explain that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
- Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

- Order and find absolute value of rational numbers.
  - Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret* $-3 > -7$ *as a statement that* $-3$ *is located to the right of* $-7$ *on a number line oriented from left to right.*
  - Write, interpret, and explain statements of order for rational numbers in real-world contexts *(e.g. Write* $-3°\text{C} > -7°\text{C}$ *to express the fact that* $-3°\text{C}$ *is warmer than* $-7°\text{C}$*).*
  - Define the absolute value of a rational number as its distance from $0$ on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of* $-30$ *dollars, write* $|-30| = 30$ *to describe the size of the debt in dollars.*
  - Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than* $-30$ *dollars represents a debt greater than* $30$ *dollars.*
- Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

**5. Apply and extend previous understandings of arithmetic to algebraic expressions.**

- Write and evaluate numerical expressions involving whole-number exponents.
- Write, read, and evaluate expressions in which letters stand for numbers.
  - Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract* $y$ *from* $5$*" as* $5 - y$*.*
  - Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression* $2(8 + 7)$ *as a product of two factors; view* $(8 + 7)$ *as both a single entity and a sum of two terms.*
  - Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6s^2$ *to find the volume and surface area of a cube with sides of length* $s = \frac{1}{2}$*.*
- Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression* $3(2 + x)$ *to produce the equivalent expression* $6 + 3x$*; apply the distributive property to the expression* $24x + 18y$ *to produce the equivalent*

*expression* $6(4x + 3y)$*; apply properties of operations to* $y + y + y$ *to produce the equivalent expression* $3y$*.*

- Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions* $y + y + y$ *and* $3y$ *are equivalent because they name the same number regardless of which number* $y$ *stands for.*
    - Recognize equivalence in variable expressions with repeated addition (such as $y + y + y = 3y$) and repeated multiplication (such as $y \times y \times y = y^3$) and use arithmetic operations to justify the equivalence.

**6. Reason about and solve one-variable equations and inequalities.**

- Describe solving an equation or inequality as a process of answering a question: Which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; recognize that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
- Solve real-world and mathematical problems by writing and solving equations of the form $x \pm p = q$ and $px = q$ for cases in which $p$, $q$ and $x$ are all nonnegative rational numbers.
- Write an inequality of the form $x > c$, $x \geq c$, $x < c$, or $x \leq c$ to represent a constraint or condition in a real-world or mathematical problem. Show that inequalities of the form $x > c$, $x \geq c$, $x < c$, or $x \leq c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

**7. Represent and analyze quantitative relationships between dependent and independent variables.**

- Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation* $d = 65t$ *to represent the relationship between distance and time.*
    - How can you determine if a variable is the independent variable or the dependent variable?
    - What are the advantages of showing the relationship between an independent and dependent variable in multiple representations (table, graph, equation)?

**8. Develop understanding of statistical variability.**

- Identify a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*
- Demonstrate that a set of data collected to answer a statistical question has a distribution that can be described by its center, spread, and overall shape.
- Explain that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

**9. Summarize and describe distributions.**

- Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
- Summarize numerical data sets in relation to their context, such as by:
    - Reporting the number of observations.
    - Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
    - Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
    - Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

**10. Geometry: Solve real-world and mathematical problems involving area, surface area, and volume.**

- Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
- Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
- Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
- Represent three-dimensional figures using nets made up of rectangles and triangles and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.